LESSON 5. 6b

More with Inverse Functions

Today you will:

- Find the inverse of radical functions
- Learn how to tell if two functions are inverses of each other
- Practice using English to describe math processes and equations

Quick review

Inverse function:

- A function found by swapping the inputs and outputs (domain/range) of the original function.
- The "reverse" or "opposite" of the original function
- A function that "undoes" the original function

Process to find the inverse of a function:

1. Rewrite original function as $y =$
2. Swap x and y
3. Solve for y

Shorthand notation for "inverse of a function"

Given $f(x)$, its inverse is written as $f^{-1}(x)$

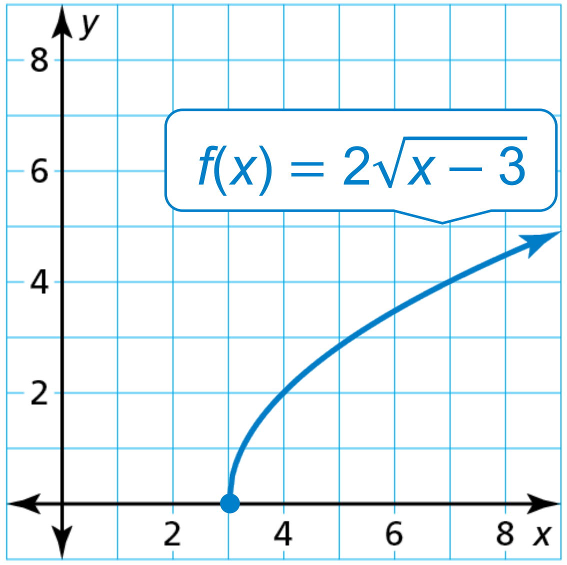
- Note: This does ***NOT*** mean $\frac{1}{f(x)}$

$f^{-1}(x)$ means the inverse of $f(x)$

Consider the function $f(x) = 2\sqrt{x-3}$. Determine whether $f^{-1}(x)$ is a function. Then find $f^{-1}(x)$.

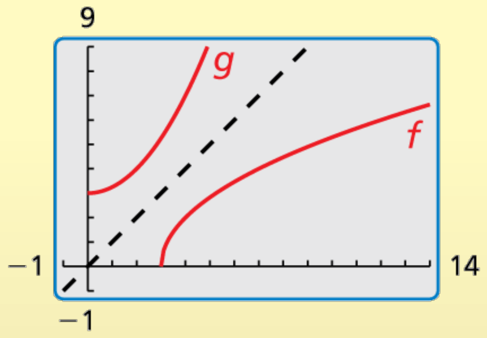
SOLUTION

Graph the function f. Notice that no horizontal line intersects the graph more than once. So, the inverse of f is a function. Find the inverse.



$y = 2\sqrt{x-3}$	Set y equal to $f(x)$.
$x = 2\sqrt{y-3}$	Switch x and y.
$x^2 = \left(2\sqrt{y-3}\right)^2$	Square each side.
$x^2 = 4(y-3)$	Simplify.
$x^2 = 4y - 12$	Distributive Property
$x^2 + 12 = 4y$	Add 12 to each side.
$\frac{1}{4}x^2 + 3 = y$	Divide each side by 4.

Check



Because the range of f is $y \geq 0$, the domain of the inverse must be restricted to $x \geq 0$.

So, the inverse of f is $g(x) = \frac{1}{4}x^2 + 3$, where $x \geq 0$.

Recall:

1. The graph of a function is its reflection around the line $y = x$.
2. The inverse of something "undoes" it

If you plug the inverse of a function into the original function, the result will be x.

Given $f(x)$ and $g(x)$,

- $f\big(g(x)\big) = x$
- $g\big(f(x)\big) = x$

This gives us a way of testing if two functions are inverses of each other:

1. Plug one into the other and evaluate/simplify
2. Do it the other way too (you must do it both ways!)
3. If the result of both is x then they are inverses of each other.

Verify that $f(x) = 3x - 1$ and $g(x) = \frac{x+1}{3}$ are inverse functions.

SOLUTION

Step 1 Show that $f(g(x)) = x$.

$$f(g(x)) = f\left(\frac{x+1}{3}\right)$$

$$= 3\left(\frac{x+1}{3}\right) - 1$$

$$= x + 1 - 1$$

$$= x \checkmark$$

Step 2 Show that $g(f(x)) = x$.

$$g(f(x)) = g(3x - 1)$$

$$= \frac{(3x-1)+1}{3}$$

$$= \frac{3x}{3}$$

$$= x \checkmark$$

Find the inverse of the function that represents the surface area of a sphere, $S = 4\pi r^2$. Then find the radius of a sphere that has a surface area of 100π square feet.

SOLUTION

Switching the variables to find the inverse would create confusion by switching the meanings of S and r. So, when finding the inverse, solve for r without switching the variables.

Step 1 Find the inverse of the function.

$$S = 4\pi r^2$$

$$\frac{S}{4\pi} = r^2$$

$$\sqrt{\frac{S}{4\pi}} = r$$

The radius r must be positive, so disregard the negative square root.

Step 2 Evaluate the inverse when $S = 100\pi$.

$$r = \sqrt{\frac{100\pi}{4\pi}}$$

$$= \sqrt{25} = 5$$

The radius of the sphere is 5 feet.

Homework

Pg 282, #37-60